Estimating the maximum possible earthquake magnitude using extreme value methodology

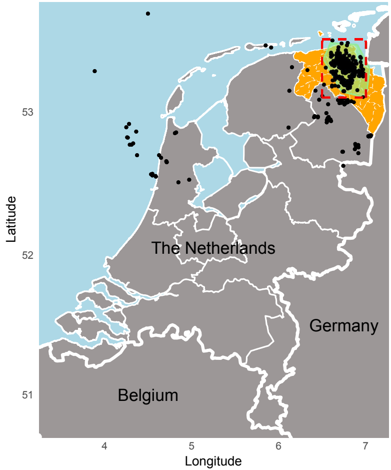
The Groningen case

Tom Reynkens[1] Jan Beirlant[1,2] Andrzej Kijko[3] John Einmahl[4]

[1] LRisk, KU Leuven, Belgium [2] University of the Free State, South Africa
[3] University of Pretoria Natural Hazard Centre, South Africa
[4] CentER, Tilburg University, The Netherlands

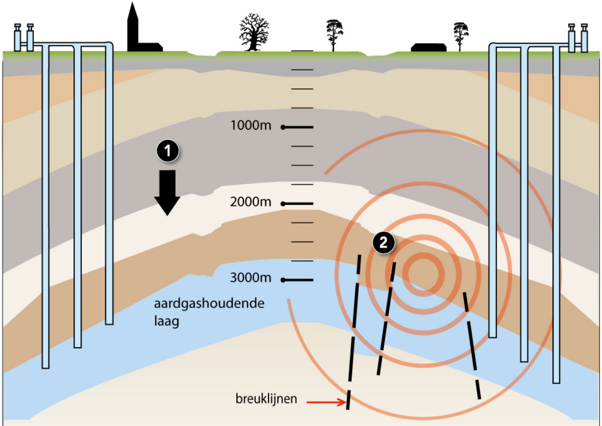
Groningen earthquakes

- \+ One of the largest gas fields in the world (2800 billion cubic metres).
- \+ Large profits for Dutch government.
- – Gas extraction induces earthquakes in the northern part of the Netherlands.
- – Damage to houses, declining house prices, etc.
- ⇒ Production lowered to 21.6 bcm/year.



Source: https://www.knmi.nl/kennis-en-datacentrum/dataset/aardbevingscatalogus

Groningen: underground



1 Door gaswinning daalt de druk in de aardgashoudende laag. Daardoor wordt deze laag langzaam in elkaar gedrukt.

2 Langs breuklijnen in de aardgashoudende laag ontstaan drukverschillen, waardoor delen van de laag plotseling kunnen verzakken. Zo ontstaan de bevingen.

© DvhN 09022013 | AB | Bron: ANP

Magnitudes and energy

Relation between earthquake magnitudes (Richter scale) and seismic energy at the epicentre (in MJ):

$$M = \frac{\log_{10}\left(\frac{E}{2}\right)}{1.5} + 1 = \frac{\ln\left(\frac{E}{2}\right)}{1.5\ln 10} + 1.$$

- High intensities possible for low magnitude earthquakes since shallow origin (3 km depth).

Maximum possible earthquake magnitude T_M

The maximum magnitude of an earthquake that can be generated by the geological structure of the area (Sintubin, 2016).

- Only depends on tectonic properties.
- Independent of evolution of seismic activity.
- Worst-case damage estimates.
- Crucial element of magnitude models.

Estimating the endpoint

1. **Parametric** estimators based on truncated Gutenberg-Richter (GR) distribution (Kijko and Sellevoll, 1989; Pisarenko et al., 1996).
2. **Non-parametric** estimators as discussed in geophysical literature (Kijko and Singh, 2011).
3. **EVT** estimators:
   - Truncated Pareto (Aban et al., 2006; Beirlant et al., 2016).
   - Truncated GPD (Beirlant et al., 2017).

- Upper confidence bounds for endpoint to quantify uncertainty of endpoint estimates.

Parametric model: Gutenberg-Richter

Truncated Gutenberg-Richter (GR) distribution (Gutenberg and Richter, 1956; Page, 1968)

Doubly truncated exponential distribution:

- $Y \sim Exp(\beta)$.
- Observe realisations of M with

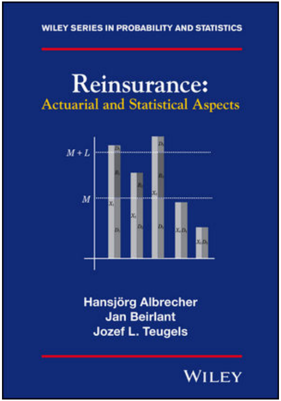
$$M =_d (Y \,|\, t_M < Y < T_M).$$

$\Rightarrow$ Distribution of M is bounded between $t_M > 0$ and T_M.

- Based on empirical evidence.
- Relationship with earthquake physics (Scholz, 1968; Scholz, 2015; Rundle, 1989).

- Extreme events:
  - Large insurance losses.
  - Financial losses.
  - Natural catastrophes: floods, earthquakes.
- Framework to deal with extreme events to compute
  - Large quantiles.
  - Return periods.
  - Small exceedance probabilities.
  - Endpoints of distributions.

- (Main) **R** packages related to EVT:
  - *actuar* (Dutang et al., 2008)
  - *evd* (Stephenson, 2002)
  - *evir* (Pfaff and McNeil, 2012)
  - *fExtremes* (Würtz and Rmetrics Association, 2013)
  - *QRM* (Pfaff and McNeil, 2016)
- CRAN task view **"Extreme Value Analysis"**.



ReIns package (Reynkens and Verbelen, 2017)

- Basic extreme value theory (EVT) estimators and graphical methods (Beirlant et al., 2004).
- EVT estimators and graphical methods adapted for censored and/or truncated data.
- Risk measures such as Value-at-Risk (VaR) and Conditional Tail Expectation (CTE).
- \+ Unified framework for all estimators and plots.

EVT estimators for endpoint of M

Upper truncation: realisations of M are observed with

$$M =_d (Y \mid Y < T_M).$$

M

Upper truncation: realisations of M are observed with

$$M =_d (Y \mid Y < T_M).$$

$$M \xrightarrow{E = e^{\ln 2 + (M-1)1.5 \ln 10}} E$$

Upper truncation: realisations of M are observed with

$$M =_d (Y \,|\, Y < T_M).$$

$$\begin{tikzcd}
M \arrow[rr, "E = e^{\ln 2 + (M-1) 1.5 \ln 10}"] & & E \arrow[d, "\substack{\text{Truncated Pareto} \\ \frac{E}{t} \,|\, E > t}"] \\
& & \hat{T}_{E,+}
\end{tikzcd}$$

Upper truncation: realisations of M are observed with

$$M =_d (Y \mid Y < T_M).$$

$$\begin{tikzcd}
M \arrow[rr, "E = e^{\ln 2 + (M-1)1.5 \ln 10}"] & & E \arrow[d, "\text{Truncated Pareto} \; \frac{E}{t} \mid E > t"] \\
\hat{T}_{M,+} & & \hat{T}_{E,+} \arrow[ll, "M = \frac{\ln\left(\frac{E}{2}\right)}{1.5 \ln 10} + 1"']
\end{tikzcd}$$

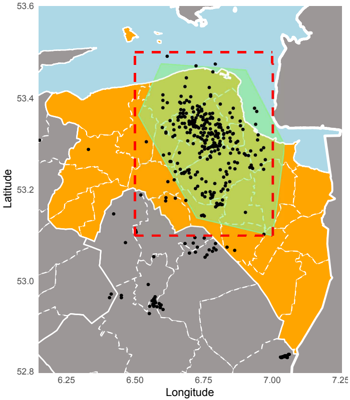
EVT estimators for endpoint of M

Upper truncation: realisations of M are observed with

$$M =_d (Y \,|\, Y < T_M).$$

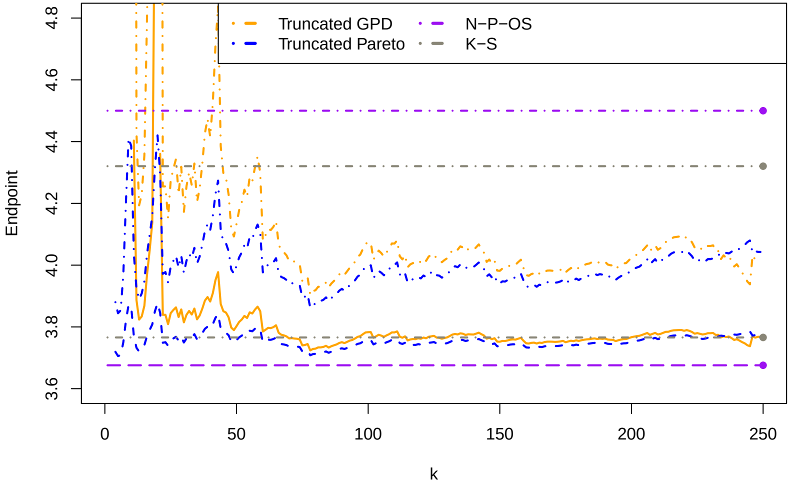
$$\begin{tikzcd}[column sep=huge, row sep=huge]
M \arrow[r, "E = e^{\ln 2 + (M-1)1.5 \ln 10}"] \arrow[d, "\begin{array}{c}\text{Truncated GPD}\\ M - t \,|\, M > t\end{array}"'] & E \arrow[d, "\begin{array}{c}\text{Truncated Pareto}\\ \frac{E}{t} \,|\, E > t\end{array}"] \\
\hat{T}_M \,|\, \hat{T}_{M,+} & \hat{T}_{E,+} \arrow[l, "M = \frac{\ln\left(\frac{E}{2}\right)}{1.5 \ln 10} + 1"']
\end{tikzcd}$$

- 286 earthquakes with magnitudes larger than $t_M = 1.5$ between December 1986 and 31 December 2016.
- Uniform noise $U[-0.05, 0.05]$ added since rounded up to one decimal digit.
- 250 smoothed magnitudes larger than $t_M = 1.5$.
- $t_M = 1.5$ is standard for Groningen (Dost et al., 2013).



Source: https://www.knmi.nl/kennis-en-datacentrum/dataset/aardbevingscatalogus

Groningen: estimates of T_M



- $t = M_{n-k,n}$: the $(k+1)$-th largest observation.
- $M_{n,n} =$ **3.6** (Huizinge, August 2012).

Outlook

- EVT-based methods perform well when estimating endpoint.
- EVT-based methods can also be used to compute large quantiles, small exceedance probabilities, etc.
- Paper is accepted in *Natural Hazards*, available on arXiv:1709.07662.

- Functions implemented in **R** package `ReIns`.
- Shiny app: https://treynkens.shinyapps.io/Groningen_app/.

Questions?

Aban, I. B., Meerschaert, M. M. and Panorska, A. K. (2006). Parameter Estimation for the Truncated Pareto Distribution. *J. Amer. Statist. Assoc.*, **101**(473), 270–277.

Albrecher, H., Beirlant, J. and Teugels, J. (2017). *Reinsurance: Actuarial and Statistical Aspects*. John Wiley & Sons, Ltd, Chichester, UK.

Beirlant, J., Fraga Alves, I. and Reynkens, T. (2017). Fitting Tails Affected by Truncation. *Electron. J. Stat.*, **11**(1), 2026–2065.

Beirlant, J., Fraga Alves, M. I. and Gomes, M. I. (2016). Tail Fitting for Truncated and Non-Truncated Pareto-Type Distributions. *Extremes*, **19**(3), 429–462.

Beirlant, J., Goegebeur, Y., Teugels, J. and Segers, J. (2004). *Statistics of Extremes: Theory and Applications*. Wiley, Chichester.

Beirlant, J., Kijko, A., Reynkens, T. and Einmahl, J. H. (2018). Estimating the maximum possible earthquake magnitude using extreme value methodology: the Groningen case. *Natural Hazards* URL: https://doi.org/10.1007/s11069-017-3162-2, accepted.

Dost, B., Caccavale, M., van Eck, T. and Kraaijpoel, D. (2013). *Report on the Expected PGV and PGA Values for Induced Earthquakes in the Groningen Area*. URL: https://www.rijksoverheid.nl/documenten/rapporten/2014/01/17/rapport-verwachte-maximale-magnitude-van-aardbevingen-in-groningen, KNMI report; last accessed on 21/02/2017.

Dutang, C., Goulet, V. and Pigeon, M. (2008). actuar: An R package for Actuarial Science. *J. Stat. Softw.*, **25**(7), 1–37.

Gutenberg, B. and Richter, C. F. (1956). Earthquake Magnitude, Intensity, Energy and Acceleration. *Bull. Seismol. Soc. Am.*, **46**(2), 105–145.

Kijko, A. and Sellevoll, M. (1989). Estimation of Earthquake Hazard Parameters From Incomplete Data Files. Part I. Utilization of Extreme and Complete Catalogs With Different Threshold Magnitudes. *Bull. Seism. Soc. Am.*, **79**(3), 645–654.

Kijko, A. and Singh, M. (2011). Statistical Tools for Maximum Possible Earthquake Estimation. *Acta Geophys.*, **59**(4), 674–700.

Page, R. (1968). Aftershocks and Microaftershocks of the Great Alaska Earthquake of 1964. *Bull. Seismol. Soc. Am.*, **58**(3), 1131–1168.

Pfaff, B. and McNeil, A. (2012). *evir: Extreme Values in R*. URL: https://CRAN.R-project.org/package=evir, R package version 1.7-3.

Pfaff, B. and McNeil, A. (2016). *QRM: Provides R-Language Code to Examine Quantitative Risk Management Concepts*. URL: https://CRAN.R-project.org/package=QRM, R package version 0.4-13.

Pisarenko, V. F., Lyubushin, A. A., Lysenko, V. B. and Golubeva, T. V. (1996). Statistical Estimation of Seismic Hazard Parameters: Maximum Possible Magnitude and Related Parameters. *Bull. Seismol. Soc. Am.*, **86**(3), 691–700.

Reynkens, T. and Verbelen, R. (2018). *ReIns: Functions from "Reinsurance: Actuarial and Statistical Aspects"*. URL: https://CRAN.R-project.org/package=ReIns, R package version 1.0.7.

Rundle, J. B. (1989). Derivation of the complete Gutenberg-Richter magnitude-frequency relation using the principle of scale invariance. *J. Geophys. Res.: Solid Earth*, **94**(B9), 12337–12342.

Scholz, C. H. (1968). The Frequency-Magnitude Relation of Microfracturing in Rock and its Relation to Earthquakes. *Bull. Seismol. Soc. Am.*, **58**(1), 399–415.

Scholz, C. H. (2015). On the stress dependence of the earthquake b value. *Geophys. Res. Lett.*, **42**(5), 1399–1402.

Sintubin, M. (2016). *De Mmax van Groningen*. URL: https://earthlymattersblog.wordpress.com/2016/08/12/de-mmax-van-groningen/, in Dutch; last accessed on 27/02/2017.

Stephenson, A. G. (2002). evd: Extreme Value Distributions. *R News*, **2**(2), 31–32.URL: https://CRAN.R-project.org/doc/Rnews/.

References IV

Würtz, D. and Rmetrics Association (2013). *fExtremes: Rmetrics - Extreme Financial Market Data*. URL: https://CRAN.R-project.org/package=fExtremes, R package version 3010.81.

Truncated Gutenberg-Richter (GR) distribution (Gutenberg and Richter, 1956; Page, 1968)

$$F_M(m) = \begin{cases} 0 & \text{if } m \le t_M \\ \frac{F_{Exp(\beta)}(m) - F_{Exp(\beta)}(t_M)}{F_{Exp(\beta)}(T_M) - F_{Exp(\beta)}(t_M)} & \text{if } t_M < m < T_M \\ 1 & \text{if } m \ge T_M \end{cases}$$

- $t_M > 0$: minimum possible magnitude
- $T_M > t_M$: maximum possible magnitude
- $\beta > 0$: rate parameter

- Doubly truncated exponential distribution.
- Based on empirical evidence.
- Relationship with earthquake physics (Scholz, 1968; Scholz, 2015; Rundle, 1989).

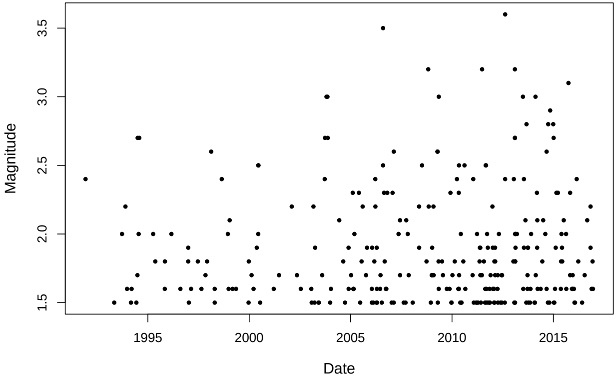


Figure: Time plot of induced earthquakes in Groningen with magnitudes larger than 1.5 in the considered area.